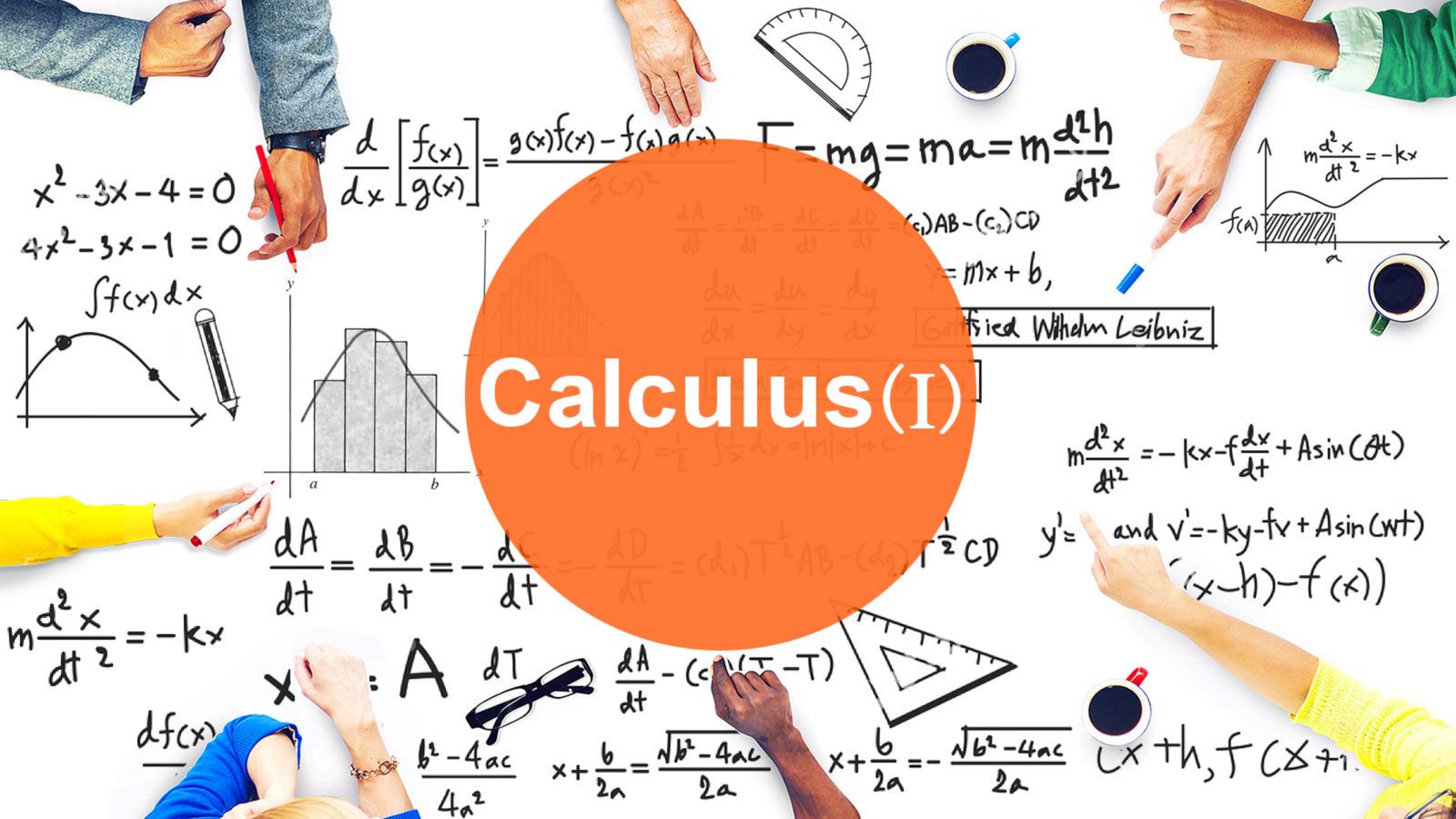


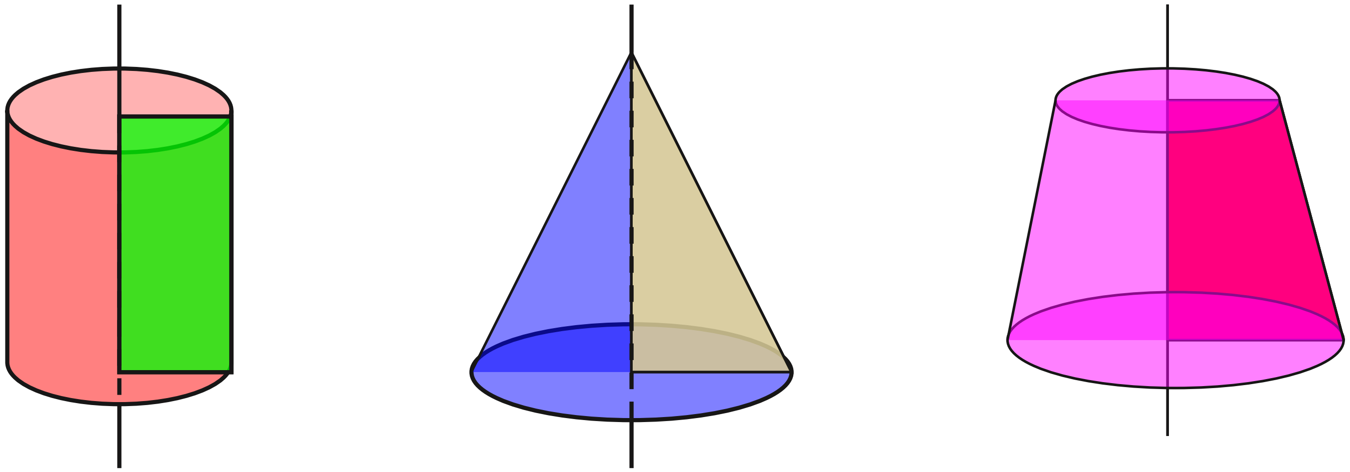
Calculus(I)

Volumes of Solids （Disk Method）

Lecturer: Xue Deng

The volume of a body of revolution

The revolver: A plane figure revolve 360-degrees of a line, it will became a 3-d graphics , the line calls the axis of rotation .



cylindrical

cone

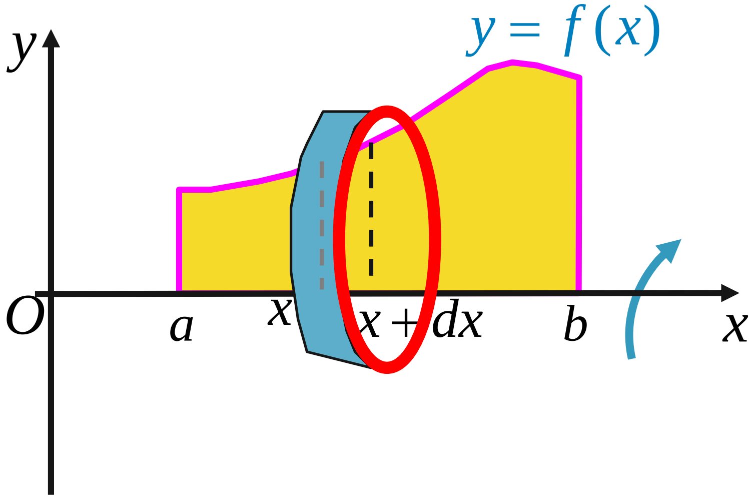
frustum of a cone

Volume of Solids (Disk Method)

Curved trapezoid around *x* –axis:

Continuous curve $y = f(x),\ x = a, x = b$ and *x*-axis.

Q:How about the volume? (*x*-style)



Element method:

1)Integration variable *x*, $x \in [a,b]$

2)$[x, x + \mathrm{d}x]$, volume element:

$$\mathrm{d}V = \pi [f(x)]^2 dx$$

3)Volume is: $V = \int_a^b \pi [f(x)]^2 \mathrm{d}x$

Volume of Solids (Disk Method)

Curved trapezoid around *y* –axis:

Continuous curve $x = \phi(y),\ y = c, y = d$ and y-axis.

Q:How about the volume? (y-style)

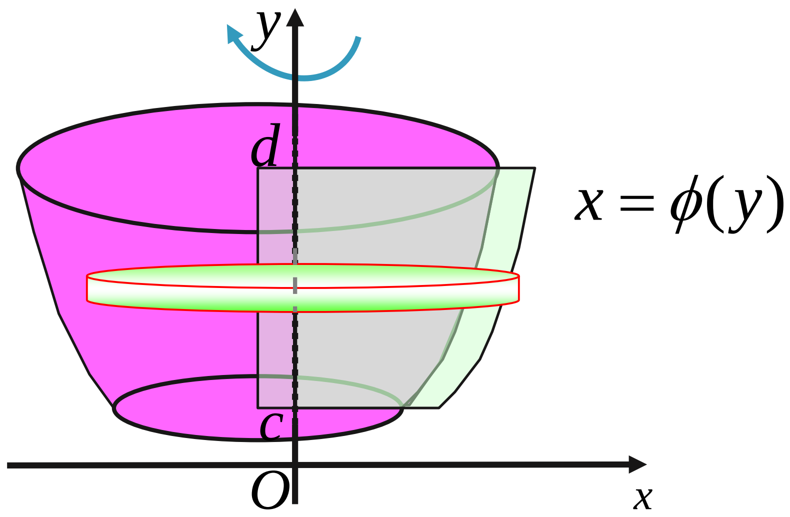
Element method:

1)Integration variable y, $y \in [c, d]$

2)$[y, y + \mathrm{d}y]$, volume element:

$$\mathrm{d}V = \pi[\phi(y)]^2 dy$$

3)Volume is: $V = \int_c^d \pi[\phi(y)]^2 \mathrm{d}y$



Example 1

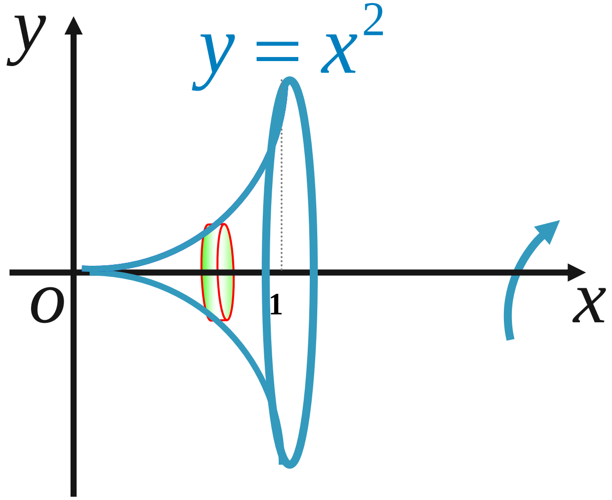
Find the volume by $y = x^2$ that revolved by $x - axis$ on $x \in [0,1]$.

Let x is integration variable, $x \in [0,1]$

$$\mathrm{d}V = \pi[f(x)]^2 \mathrm{d}x$$

volume element: $\mathrm{d}V = \pi y^2 \mathrm{d}x = \pi x^4 \mathrm{d}x$

$$V = \pi \int_0^1 x^4 dx = \frac{\pi}{5}$$



Example 2

Find the volume by $y = x^2$ and $y = \sqrt{x}$ that revolved by $y - axis$.

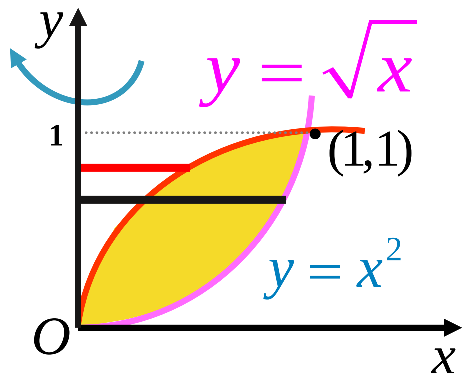
$$\begin{cases} y = x^2 \\ y = \sqrt{x} \end{cases} \Rightarrow (0,0) \text{ and } (1,1),$$

$$dV = \pi[f(y)]^2 dy$$

$$V = \pi\int_0^1 (\sqrt{y})^2 dy - \pi\int_0^1 (y^2)^2 dy$$

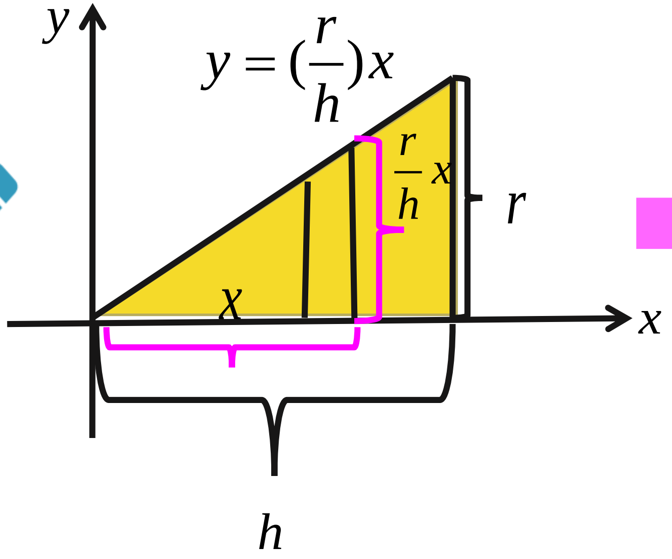
$$= \pi\int_0^1 (y - y^4) dy$$

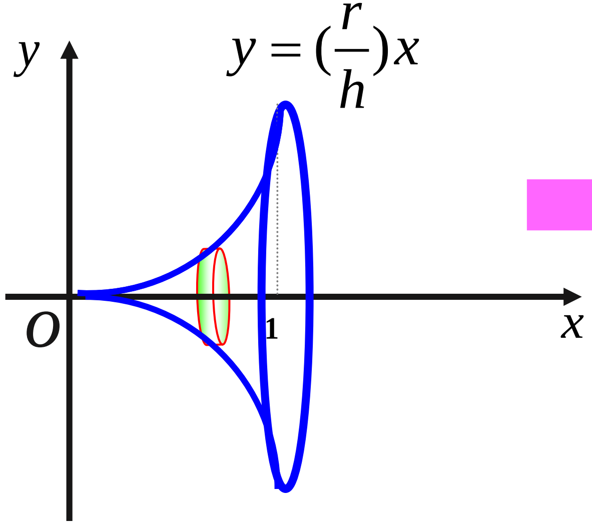
$$= \frac{3}{10}\pi$$



Example 3

The region bounded by the line $y=(\frac{r}{h})x, x\text{-}axis, x=h$ is revolved about the $x\text{-}axis$ there by generated a core$(r>0, h>0)$, find its volume by the disk method.





$$V=\pi\int_0^h \frac{r^2}{h^2}x^2\mathrm{d}x$$

$$=\pi\frac{r^2}{h^2}\int_0^h x^2\mathrm{d}x$$

$$=\pi\frac{r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h$$

$$=\frac{\pi r^2 h}{3}.$$

Questions and Answers

Q1: Set up and evaluate an integral for the volume of the solid that results when the region R shown in folwing figure is revolved about x-axis.

$$\Delta V \approx \pi(3+2x-x^2)^2\Delta x,$$

$$V=\pi\int_0^3(3+2x-x^2)^2\mathrm{d}x.$$

$$V=\pi\int_0^3(3+2x-x^2)^2\mathrm{d}x$$

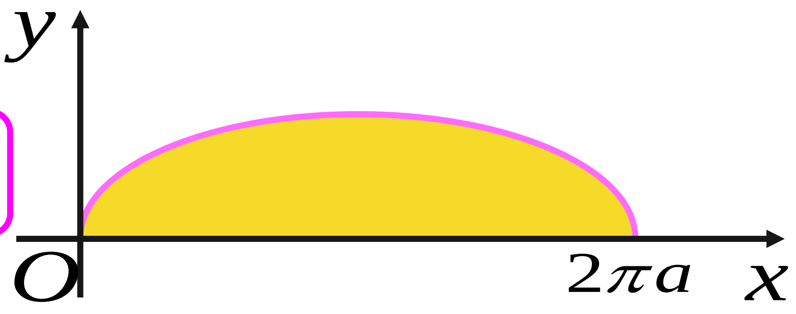
$$=\frac{153}{5}\pi.$$

Questions and Answers

Q2: Find the different volume by $x = a(t - \sin t)$, $y = a(1 - \cos t)$ and $y = 0$ that revolved by $y-$axis and $x-$axis.

① Case1:revolved by x-axis

$x = a(t - \sin t)$



$$V_x = \int_0^{2\pi a} \pi y^2(x)\mathrm{d}x$$

$$= \pi \int_0^{2\pi} a^2(1-\cos t)^2 \cdot a(1-\cos t)\mathrm{d}t$$

$$= \pi a^3 \int_0^{2\pi} (1 - 3\cos t + 3\cos^2 t - \cos^3 t)\mathrm{d}t$$

$$= 5\pi^2 a^3.$$

2 Case2:revolved by y-axis

The volume can be computed by volume of OABC revolved by y-axis subtraction volume of OBC revolved by y-axis.

cycloid: $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$

$$V_y = \int_{0(A)}^{2a(B)} \pi x_2^2(y)\mathrm{d}y - \int_{0\,(O)}^{2a\,(B)} \pi x_1^2(y)\mathrm{d}y$$

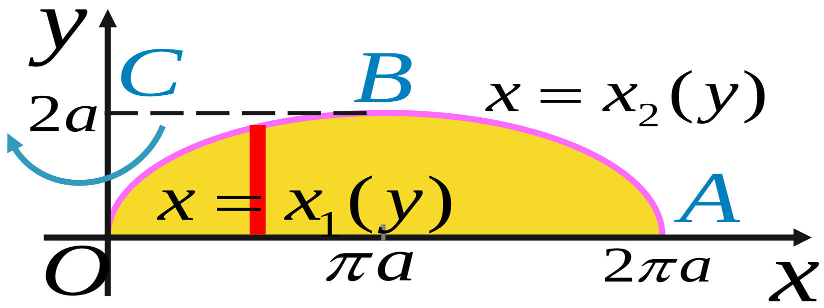
$$= \pi \int_{2\pi}^{\pi} a^2(t - \sin t)^2 \cdot a \sin t\mathrm{d}t - \pi \int_{0}^{\pi} a^2(t - \sin t)^2 \cdot a \sin t\mathrm{d}t$$

$$= -\pi a^3 \int_0^{2\pi} (t - \sin t)^2 \sin t\mathrm{d}t$$

$$= \int_0^{2\pi} (t^2 \sin t - 2t\sin^2 t + \sin^3 t)\,\mathrm{d}t$$

$$= 6\pi^3 a^3.$$

let $u = t - \pi$



Volumes of Solids
（Disk Method）